

Physics 1A Discussion (Week 6)

Problem 1 (problem 5.76 on page 165 of the textbook)

Block A in Fig weighs 60.0 N. The coefficient of static friction between the block and the surface on which it rests is 0.25. The weight w is 12.0 N, and the system is in equilibrium.

(a) Find the friction force exerted on block A.

The situation for this system is shown by Fig 1.

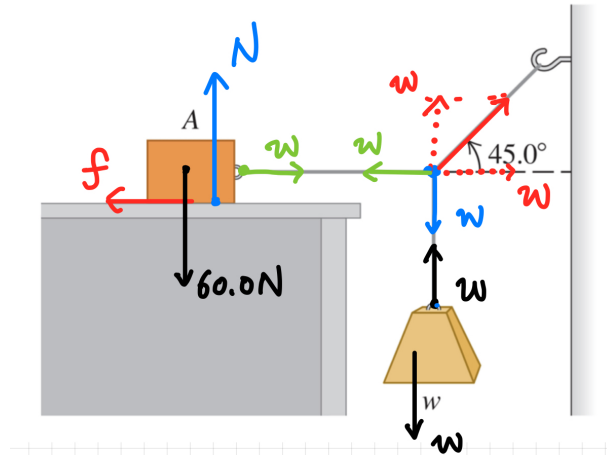


Figure 1: Caption

The force equilibrium in horizontal direction with respect to the object A implies

$$f = w = 12.0 \text{ N} \quad (1)$$

(b) Find the maximum weight w for which the system will remain in equilibrium.

$$\begin{aligned} w_{\max} &= f_{\max} \\ &= \mu N_A \\ &= 0.25 \times 60 \text{ N} \\ &= 15 \text{ N} \end{aligned} \quad (2)$$

Problem 2 (problem 5.97 on page 167 of the textbook)

Block A , with weight $3w$, slides down an inclined plane S of slope angle 36.9° at a constant speed while plank B , with weight w , rests on top of A . The plank is attached by a cord to the wall.

(a) Draw a diagram of all the forces acting on block A .

The relevant forces acting on A are

- Weight (Gravity) of A : $3w$
- Friction forces from B : f_B
- Friction forces from S : f_S
- Normal forces from B : N_B
- Normal forces from S : N_S

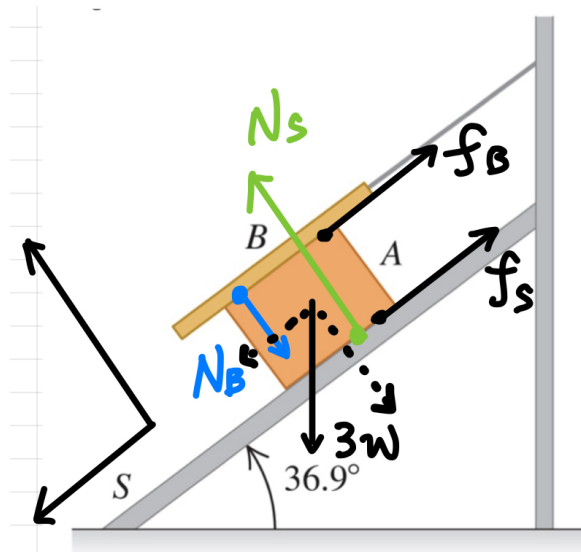


Figure 2: Force diagram with respect to the object A .

(b) If the coefficient of kinetic friction is the same between A and B and between S and A , determine its value. We solve this for μ

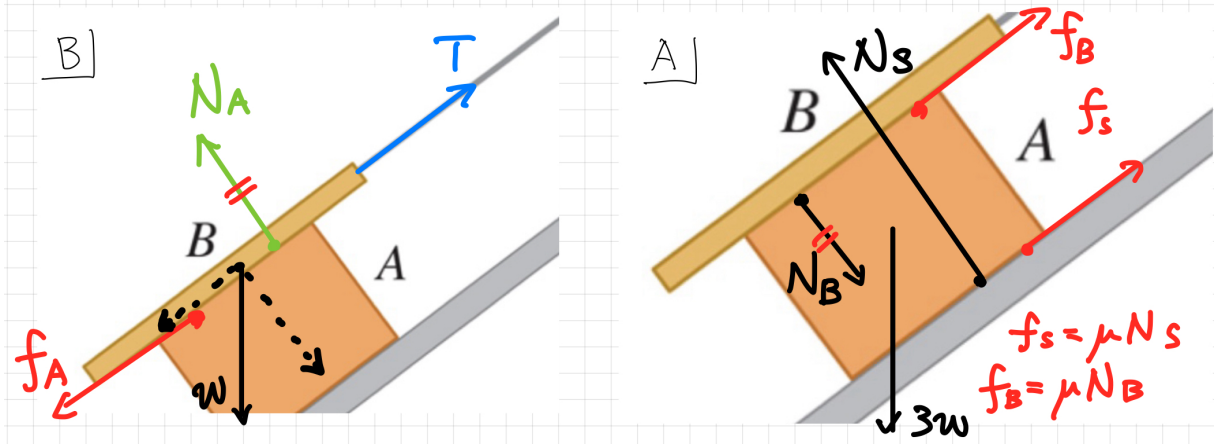


Figure 3: (Left) Force diagram with respect to the object B, (Right) Force diagram with respect to the object A

Let μ be the friction coefficient, and $\theta = 36.9^\circ$. Then the force equilibrium with respect to the object A is given by

$$\text{Horizontal: } 3w \sin \theta - \mu N_B - \mu N_S = 0, \quad (3)$$

$$\text{Vertical: } N_S - N_B - 3w \cos \theta = 0, \quad (4)$$

and the force equilibrium with respect to the object B is given by

$$\text{Horizontal: } f_A + w \sin \theta - T = 0, \quad (T: \text{ Tension force}) \quad (5)$$

$$\text{Vertical: } N_A - w \cos \theta = 0. \quad (6)$$

From the action-reaction principle, we know that $\vec{N}_A$ and $\vec{N}_B$ have the same magnitude with opposite directions. Then we see that

$$N_A = w \cos \theta \Rightarrow N_B = N_A = w \cos \theta \quad (7)$$

$$N_S = 4w \cos \theta. \quad (8)$$

Therefore, you get

$$3w \sin \theta - \mu N_B - \mu N_S = 0 \Rightarrow \mu = \frac{3}{5} \tan \theta \simeq 0.45. \quad (9)$$

Problem 3 (problem 6.76 on page 197 of the textbook)

The spring of a spring gun has force constant $k = 400 \text{ N/m}$ and negligible mass. The spring is compressed 6.00 cm , and a ball with mass 0.0300 kg is placed in the horizontal barrel against the compressed spring. The spring is then released, and the ball is propelled out the barrel of the gun. The barrel is 6.00 cm long, so the ball leaves the barrel at the same point that it loses contact with the spring. The gun is held so that the barrel is horizontal.

(a) Calculate the speed with which the ball leaves the barrel if you can ignore friction.

The setting for part (a) is shown in Fig 4.

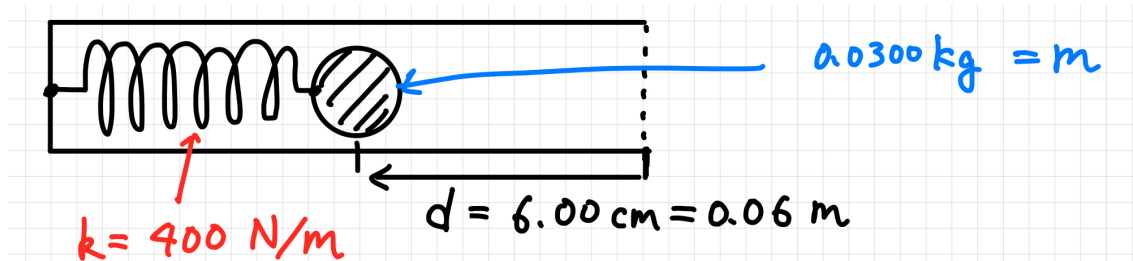


Figure 4: The setting for part (a)

When we utilize the conservation of energy:

$$(\text{Initial Total Energy}) = (\text{Total Energy at the Final Position}) \quad (10)$$

Since $(\text{Initial Total Energy}) = \frac{1}{2}kd^2 + 0 = \frac{1}{2}kd^2$, and $(\text{Total Energy at the Final Position}) = 0 + \frac{1}{2}mv_1^2$, we get

$$\begin{aligned} \frac{1}{2}kd^2 &= \frac{1}{2}mv_1^2 \\ \Rightarrow v_1 &= \sqrt{\frac{k}{m}d^2} \simeq 6.93 \text{ m/s} \end{aligned} \quad (11)$$

(b) Calculate the speed of the ball as it leaves the barrel if a constant resisting force of 6.00 N acts on the ball as it moves along the barrel.

The modified setting for part (b) is shown in Fig 5.

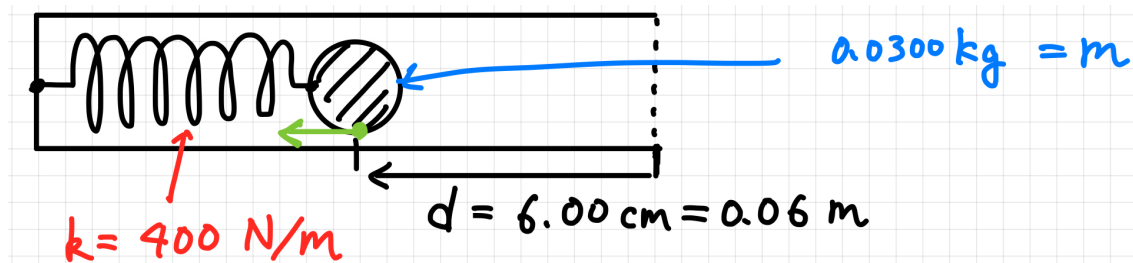


Figure 5: The modified setting for part (b)

The work W done by the resistance force $\vec{f}$ is given by

$$\begin{aligned} W &= \vec{f} \cdot \vec{d} \\ &= (-6.00 \text{ N})(0.06 \text{ m}) \\ &= -0.36 \text{ J} < 0 \text{ J}. \end{aligned} \quad (12)$$

Conservation of energy implies

$$\begin{aligned} \frac{1}{2}kd^2 + W &= \frac{1}{2}mv_2^2 \\ \Rightarrow v_2 &\simeq 4.90 \text{ m/s} < 6.93 \text{ m/s} \simeq v_1 \end{aligned} \quad (13)$$

(c) For the situation in part (b), at what position along the barrel does the ball have the greatest speed, and what is that speed? (In this case, the maximum speed does not occur at the end of the barrel.)

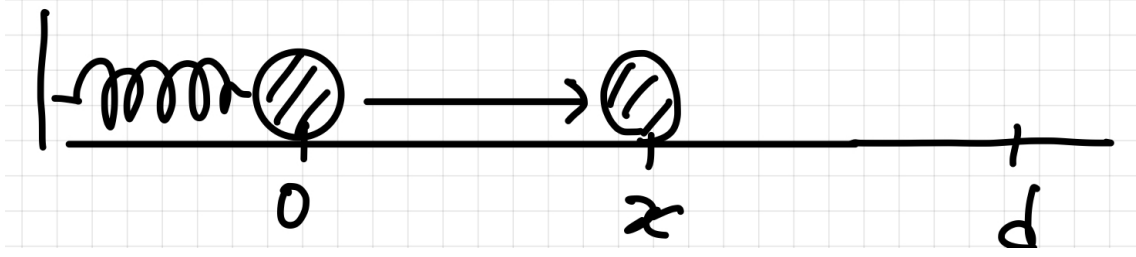


Figure 6: Schematic figure of the travel along the barrel

Potential energy release at position x ($0 \leq x \leq d$) is given by

$$\frac{1}{2}kd^2 - \frac{1}{2}k(d-x)^2 = kdx - \left(\frac{1}{2}\right)kx^2. \quad (14)$$

The work done by the friction force at position x is given by

$$W(x) = -fx \quad (15)$$

Then, the energy conservation implies

$$\begin{aligned} kdx - \left(\frac{1}{2}\right)kx^2 + W(x) &= \frac{1}{2}mv(x)^2 \\ \frac{1}{2}mv(x)^2 &= -\frac{1}{2}k\left(x - \frac{kd-f}{k}\right)^2 + \frac{(kd-f)^2}{2k}. \end{aligned} \quad (16)$$

The ball reaches at the maximum velocity at position

$$x = \frac{kd-f}{k} \sim 4.5 \text{ m}, \quad (17)$$

and the maximum velocity is

$$\begin{aligned} v_{\max} &= v\left(x = \frac{kd-f}{k}\right) \\ &= \frac{(kd-f)^2}{2k} \\ &\sim 5.2 \text{ m/s}. \end{aligned} \quad (18)$$